REPORT No. 186

THE APPLICATION OF PROPELLER TEST DATA TO DESIGN AND PERFORMANCE CALCULATIONS

By WALTER S. DIEHL
Bureau of Aeronautics
Navy Department

REPORT No. 186

THE APPLICATION OF PROPELLER TEST DATA TO DESIGN AND PERFORMANCE CALCULATIONS.

By Walter S. Diehl.

SUMMARY.

This report is a study of test data on a family of Durand's propellers (Nos. 3, 7, 11, 82, 113, 139), which is fairly representative of conventional design, prepared for publication by the National Advisory Committee for Aeronautics. The test data are so plotted that the proper pitch and diameters for any given set of conditions are readily obtained. The same data are plotted in other forms which may be used for calculating performance when the ratio of pitch to diameter is known. These new plots supply a means for calculating the performance, at any altitude, of airplanes equipped with normal or supercharged engines.

The coefficients used and the methods of plotting adopted in this report coordinate the results of a few tests into complete families of curves covering the entire range of p/D ordinarily used. This method of analyzing test data enables an investigator to plan tests systematically and leads to useful application of test data.

INTRODUCTION.

The conventional methods of plotting and tabulating propeller data are undoubtedly the most logical forms in which the test results can be presented, and they are quite satisfactory for a single propeller; but when we come to study a family of propellers in which the pitch is the only variable, new methods of plotting must be adopted if the full value of the data is to be available. All airplane designers who have had occasion to use data from Durand and Lesley's or similar tests are fully acquainted with the difficulties encountered in applying these data to design problems. This report has been prepared at the suggestion of Dr. D. W. Taylor to supply data for design and test analysis.

The family of propellers, Durand numbers 139, 11, 7, 3, 82, and 113 with nominal pitch ratios 0.3, 0.5, 0.7, 0.9, 1.1, and 1.3 respectively, was chosen as being most representative of the conventional designs. These propellers have narrow, tapered blades and a more or less conventional section with an uncambered driving face. The nominal pitch values are constant along the blade, referred to as the driving face. (This nominal pitch is frequently called "face pitch.")

The methods of plotting the data used in this report enable the engineer to solve three distinct problems and variations with very little effort and with results as accurate as the test data. These problems are:

(*a*) Given a set of conditions, $B.HP$, V, and $R.P.M.$, what pitch, p, and diameter, D, are most suited? What efficiency can be obtained? How does efficiency vary with p and D?

(*b*) With a given pitch, diameter and engine power curve, how do the efficiency, η, and HP available vary with air speed?

(*c*) With a given pitch, diameter, power-required curve, and engine power curve, how do efficiency, η, and $B.HP$ vary with air speed?

There is another feature of great importance. This method of plotting propeller data enables the investigator to plan his work so as to supply the engineer with information of value. Instead of random tests there can be a systematic investigation leading to definite results.

PITCH AND DIAMETER.

In practically all propeller design problems the engineer is required to find the pitch and diameter required to absorb a given power, P, at a given translational speed, V, and rotational speed, n. A very convenient method may be built up on the use of the nondimensional coefficient C_3 as given in National Advisory Committee Aeronautics for Technical Report No. 141 (Durand and Lesley). C_3 is defined as

$$C_3=\frac{Pn^2}{\rho V^5} \quad \text{-------- (1)}$$

where ρ is the air density and the other symbols have their usual meanings. When we study the values of C_3 for various propellers, it is found that they vary from 0.02 to 3 or more. This variation is too great for practical use. A great improvement is obtained by extracting the square root of the reciprocal

$$\sqrt{\frac{1}{C_3}}=\sqrt{\frac{\rho V^5}{Pn^2}}=\frac{V}{n}\sqrt{\frac{\rho V^3}{P}} \quad \text{-------- (2)}$$

This is equivalent to the reciprocal of the ρ function employed by Admiral Taylor; both are nondimensional factors independent of the diameter. Let the new factor be denoted by any convenient symbol, say F. At this time it is to be noted that the factor F is more suitable than ρ for propellers in that the working range is more advantageously located in the numerical scale for plotting.

The factor F has been calculated for each of the six propellers of the family under consideration in Tables I–VI. The values are plotted logarithmically as abscissae against $\frac{V}{nD}$ as ordinates, in the lower section of Figure 1. The upper section of the same figure contains

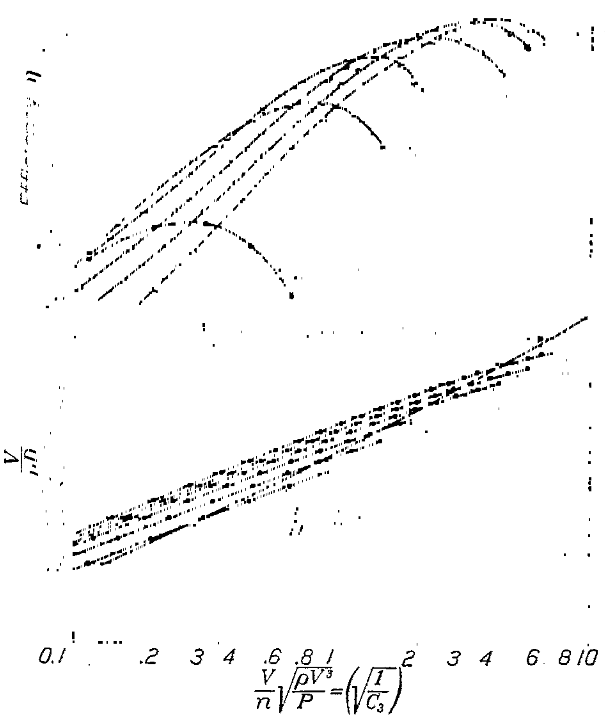

FIG. 1.—Relation between $\sqrt{\frac{1}{C_3}}$, $\frac{p}{D}$, $\frac{V}{nD}$ and efficiency. Durand propellers Nos. 139, 11, 7, 3, 82, and 113.

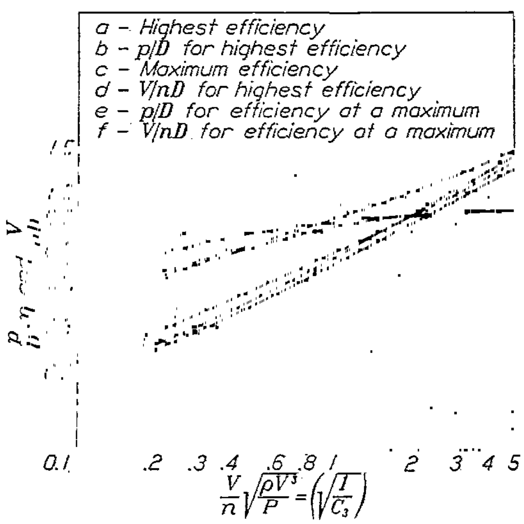

FIG. 2.—Relation between $\sqrt{\frac{1}{C_3}}$, efficiency, $\frac{V}{nD}$ and $\frac{p}{D}$. Durand propellers Nos. 139, 11, 7, 3, 82, and 113.

the corresponding values of efficiency, η, as ordinates, on a semilogarithmic plot. In this case the logarithmic scale is used largely to contract the scale to reasonable limits.

A study of Figure 1 brings out two outstanding features. The first and most important is that we have six propellers for which F, $\frac{p}{D}$, and the $\frac{V}{nD}$ corresponding to each individual maximum efficiency are known. That is, we have six values of F, each corresponding to a known value of $\frac{V}{nD}$ at which one propeller of known $\frac{p}{D}$ has its maximum efficiency. These values of $\frac{p}{D}$ and $\frac{V}{nD}$ plot as smooth curves against F as may be seen in Figure 2. These curves are the essential design curves and their use will be explained later. The other feature is well known and of minor importance although of considerable interest. At any given value of F there is but one propeller which gives its maximum efficiency for these conditions. This is the propeller having the $\frac{p}{D}$ and $\frac{V}{nD}$ previously determined. However, its efficiency is not the highest that can be obtained at the given value of F. The highest possible efficiency at each value of F is determined with $\frac{V}{nD}$ for each of the six propellers of known $\frac{p}{D}$ from the upper section of Figure 1. The values of $\frac{p}{D}$ and $\frac{V}{nD}$ so obtained are plotted on Figure 2 as broken lines to prevent confusion with the corresponding values for the maximum efficiency propellers.

The application of these curves to design is simple. The value of F is determined by the design conditions of P, V, n. Note that the values must be in consistent units. The foot-pound second system is recommended for propeller design, so that $P=550$ B. HP ft lb, $V=$ft/sec, $n=$r. p. s. and $\rho=0.00237$ slugs/ft^3 (at sea level). Using the heavy curves on Figure 2, the values of $\frac{p}{D}$ and $\frac{V}{nD}$ are found. Since $\frac{V}{n}$ is known

$$D=\frac{V}{n}\cdot\frac{nD}{V}$$

This procedure assumes that P, V, and n at which the efficiency is to be a maximum are known.

The following specimen calculation will illustrate the simplicity of the method: Assume $V=120$ $M.P.H.=176$ ft/sec, $B.HP=220$ and $R.P.M.=N=1{,}800$, or r. p. s. $=n=30$ then

$$\frac{V}{n}=5.86 \text{ and } \sqrt{\frac{\rho V^3}{P}}=\sqrt{\frac{0.00237\times 176^3}{220\times 550}}=0.320$$

$$F=\frac{V}{n}\sqrt{\frac{\rho V^3}{P}}=5.86\times 0.320=1.875$$

Figure 7 has been prepared to simplify the calculation of F. It gives the terms $\sqrt{\frac{\rho_0 V^3}{P}}$ directly in terms of V in $M.P.H.$ and P in $B.HP$ at sea level. For any given altitude the value given by Figure 7 must be multiplied by the corresponding value of $\sqrt{\frac{\rho}{\rho_0}}$. Referring to Figure 2, it is seen that the propeller having $\frac{p}{D}=0.79$ has its maximum efficiency $\eta=0.80$ at $F=1.875$, and $\frac{v}{ND}=0.73$. The diameter of this propeller is

$$D=5.86\div 0.73=8.02 \text{ ft.}$$

A diameter of 8 feet was actually used with satisfactory results on a design having the characteristics assumed in this calculation.

FULL LOAD POWER, p AND D KNOWN.

Having determined or given, p and D, a common problem is to find n, η and the maximum power available at various airspeeds. A comparatively simple method employs Durand's coefficient C_2 (National Advisory Committee for Aeronautics Technical Report No. 141) and assumes constant torque over the range of n under consideration.

The coefficient C_2 is defined as

$$C_2=\frac{P}{\rho V^3 D^2} \qquad (3)$$

Multiplying C_2 by $\frac{v}{nD}$ we obtain another coefficient which may be designated C_4

$$C_4=C_2\frac{v}{nD}=\frac{P}{\rho\, n\, V^2 D^3} \qquad (4)$$

Note that C_4, is proportional to the Q_c of Durand's earlier reports in the relation

$$C_4 = \frac{2\pi g}{1000} Q_c$$

The common formula for engine power is

$$P = 2\pi\, nQ \text{ ft/lb/sec} \qquad (5)$$

where Q is the torque in lb ft. Substituting this in equation (4) we obtain

$$C_4 = \frac{2\pi Q}{\rho V^2 D^3} \qquad (6)$$

Now Q is substantially constant and may be so assumed without serious error, or more accurate results may be obtained by estimating the probable value of Q and n for each condition, using the characteristic curves for the engine. If still greater accuracy is required, a second approximation should be sufficient to give results perhaps more accurate than the experimental data justifies. We may therefore assume that Q, V, ρ and D are known, so that C_4 is known. With a curve of C_4 against $\frac{V}{nD}$ we obtain the $\frac{V}{nD}$ and since $\frac{V}{D}$ is known, n is determined. From the characteristic curves of the engine and the propeller the corresponding $B.HP$ and propeller efficiency are found.

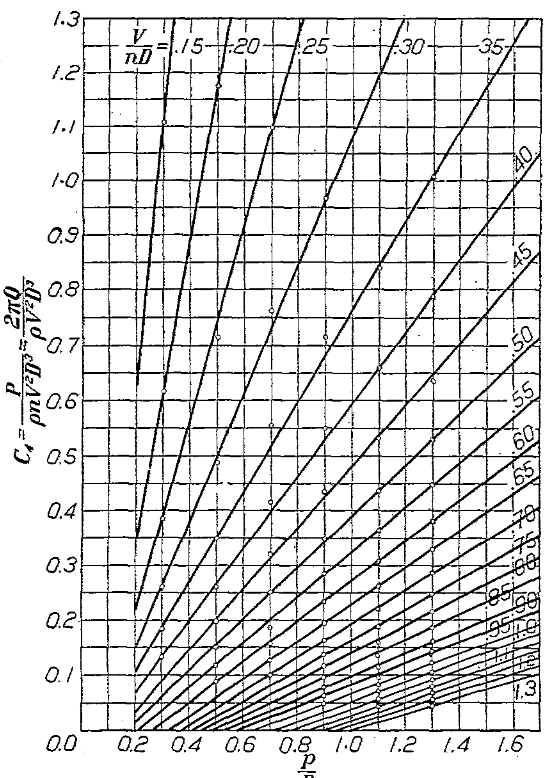

FIG. 3.—Variation of C_4 with $\frac{p}{D}$ and $\frac{V}{nD}$. Durand propellers 139, 11, 7, 3, 82, and 113.

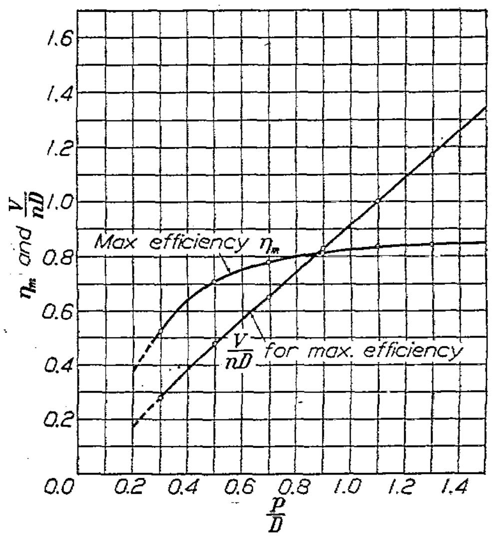

FIG. 4.—Relation between $\frac{p}{D}$, maximum efficiency η_m and $\frac{V}{nD}$ for η_m. Durand propellers Nos. 139, 11, 7, 3, 82, and 113.

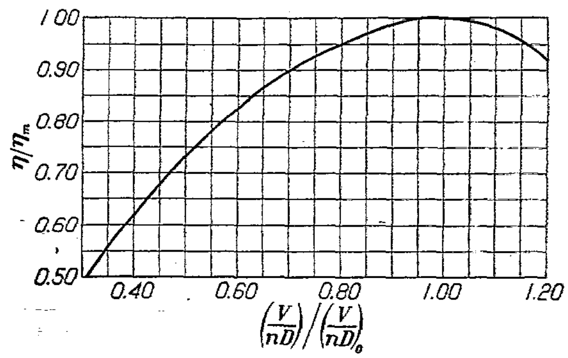

FIG. 5.—General efficiency curve for air propellers based on Durand's tests. Note: η_m the maximum efficiency, occurs when $\left(\frac{V}{nD}\right) = \left(\frac{V}{nD}\right)_0$.

The coefficient C_4 at a given $\frac{V}{nD}$ is found to plot against $\frac{p}{D}$ as a straight line as shown on Figure 3 which contains such lines for all of the data given by Durand for the family of propellers under consideration. In any given case there are two methods of using Figure 3. The better method would be to read off the values of C_4 corresponding to the $\frac{V}{nD}$'s intersected by the abscissa of the $\frac{p}{D}$ used and draw a faired curve through these values of C_4 plotted against $\frac{V}{nD}$. A quicker and somewhat less accurate method would be to estimate by interpolation from Figure 3, the value of $\frac{V}{nD}$ correspondingly to the known C_4 and $\frac{p}{D}$

In order to facilitate these calculations two additional figures, Nos. 4 and 5, have been included in this report. Figure 5 is taken from National Advisory Committee for Aeronautics Technical Report No. 168 and shows the efficiency at any $\frac{V}{nD}$ for any propeller when the maximum efficiency and the $\frac{V}{nD}$ for maximum efficiency are known. Figure 4 gives these two factors plotted against $\frac{p}{D}$, and is to be used instead of Figure 2 (which gives the same information) when $\frac{p}{D}$ is known.

The method just outlined applies particularly to calculations of maximum effective power at any given airspeed and air density. It is therefore well suited to the calculation of airplane performance at altitudes with either normal or supercharged engines. The variation of Q with n (and ρ) is the characteristic of the engine and must be known.

THROTTLED POWER, p AND D KNOWN.

In the calculations for throttled flight for any given airplane, we have known V, D and power required, HP_r. To obtain n, η, and the corresponding $B.HP$, use will again be made of the coefficient

$$C_2=\frac{P}{\rho V^3 D^2} \quad \text{(3)}$$

Multiplying C_2 by the propeller efficiency η gives

$$(\eta C_2)=\frac{\eta P}{\rho V^3 D^2} \quad \text{(7)}$$

Note that $\eta P=550\ HP_r$, so that

$$(\eta C_2)=\frac{550\ HP_r}{\rho V^3 D^2} \quad \text{(7a)}$$

The values of ηC_2 for the family of propellers under consideration are calculated in Tables I–VI, and plotted in Figure 6, as ordinates against $\frac{p}{D}$ as abscissae with lines of constant $\frac{V}{nD}$. For propellers of low $\frac{p}{D}$ ratio this plot is satisfactory, but for high ratios of $\frac{p}{D}$ and $\frac{V}{nD}$ the values of ηC_2 become too small to be read off accurately. In order to remedy this condition Figure 7 has been prepared with $\sqrt{\eta C_2}$ instead of ηC_2 as ordinates. This operation contracts the variation in the ordinate to a range within which accurate readings may be made.

The use of these two figures is almost self-explanatory. Given a curve of HP_r *vs.* airspeed, the values of ηC_2 are calculated for appropriate or desired airspeeds (using Equation 7a). From Figure 6 or Figure 7 according to $\frac{V}{nD}$ range and accuracy required, the values of $\frac{V}{nD}$ corresponding to each value of ηC_2, may be estimated on the vertical of the $\frac{p}{D}$. More accurate results could be obtained by constructing a curve of ηC_2 vs. $\frac{V}{nD}$ for the desired $\frac{p}{D}$. Having obtained the $\frac{V}{nD}$ at each V, n and consequently $B.HP$ and η are known.

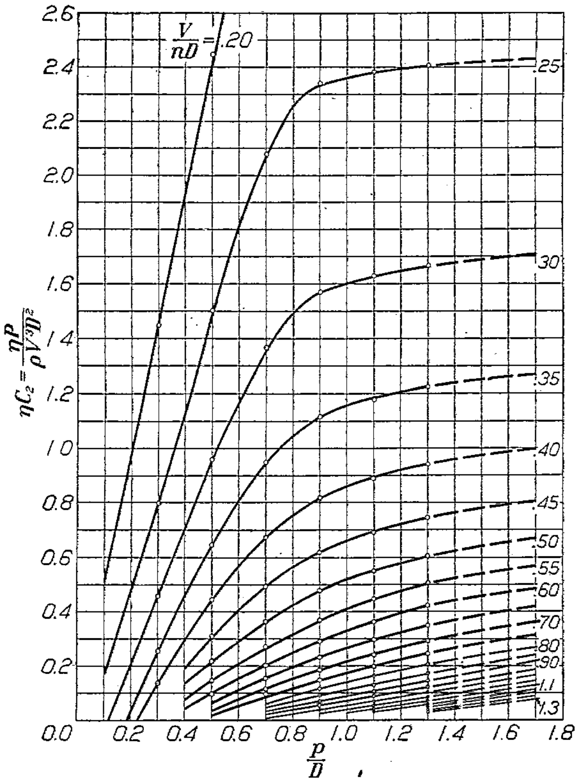

FIG. 6.—Variation of ηC_2 with $\frac{p}{D}$ and $\frac{V}{nD}$. Durand propellers Nos. 139, 11, 7, 3, 82, and 113.

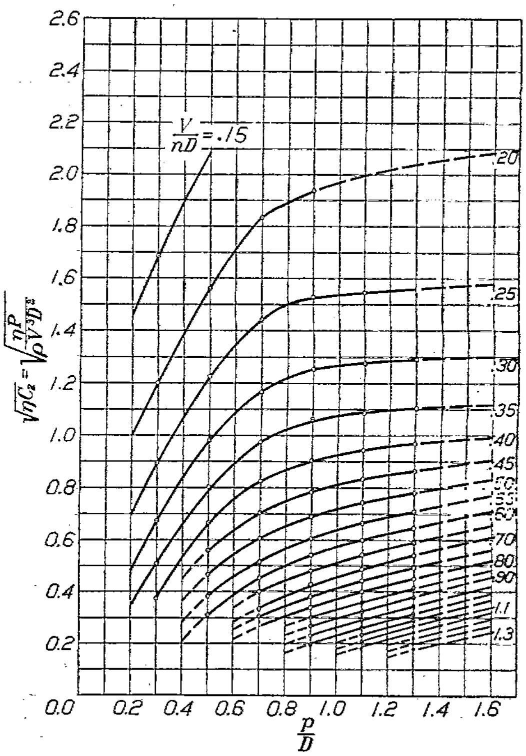

FIG. 7.—Variation of $\sqrt{\eta C_2}$ with $\frac{p}{D}$ and $\frac{V}{nD}$. Durand's propellers Nos. 139, 11, 7, 3, 82, and 113.

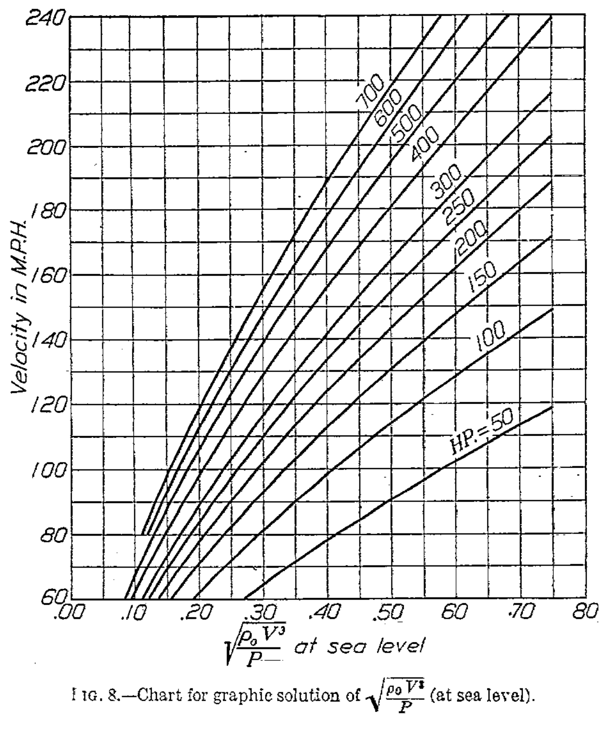

FIG. 8.—Chart for graphic solution of $\sqrt{\frac{\rho_0 V^3}{P}}$ (at sea level).

COMMENT.

The application of test data to actual design of propellers has not been given sufficient trial to enable one to judge the reliability either of the test data or of these methods of application. In the few cases calculated for comparison by the writer, the test data gave very consistent results. This may have been the result of coincidences. The use of these data and methods must therefore depend on the results of further comparisons with conventional designs.

The two most important features of this study are the applications to performance calculations and the guide furnished to the investigator concerning data required by the engineer. In regard to the latter it appears that a few well chosen tests on propellers of conventional and proved designs should supply complete design and engineering data, when the results are plotted in the general form adopted in this report. It may be found later that other coefficients and methods of plotting yield even better results.

From a study of the curves included in this report, it may be concluded that the tests now most needed are three series of varying $\frac{p}{D}$ giving three variations in the blade width, or aspect ratio, to cover the usual design variation. In these tests a proved blade form should be used, the variations in blade width obtained by proportional changes, in so far as this is practical. The camber ratio at each radius should be held constant at the values determined by the usual design or empirical curves of "minimum camber ratio."

TABLE I.

DURAND PROPELLER NO. 139.

$$\frac{p}{D}=0.3$$

$\frac{V}{nD}$	η	C_2	C_3	$\sqrt{\frac{1}{C_3}}$ F	$\frac{V}{nD}C_2$ C_4	ηC_2	$\sqrt{\eta C_2}$
0.15	0.385	7.3800	328.000	0.0548	1.1070	2.8420	1.687
.20	.469	3.0880	77.200	.1140	.6170	1.4480	1.202
.25	.517	1.5420	24.680	.2010	.3860	.7980	.893
.30	.522	.8667	9.630	.3220	.2602	.4525	.673
.35	.490	.5248	4.825	.4830	.1837	.2572	.507
.40	.415	.3343	2.090	.6920	.1337	.1387	.373

$$C_2=\frac{P}{\rho V^3 D^2}$$

$$C_3=\frac{Pn^2}{\rho V^3}$$

TABLE II.

DURAND PROPELLER NO. 11.

$$\frac{p}{D}=0.5$$

$\frac{V}{nD}$	η	C_2	C_3	$\sqrt{\frac{1}{C_3}}$ F	$\frac{V}{nD}C_2$ C_4	ηC_2	$\sqrt{\eta C_2}$
0.20	0.434	5.6380	141.000	0.0842	1.1270	2.445	1.5640
.25	.522	2.8550	45.670	.1480	.7138	1.505	1.2270
.30	.590	1.6300	18.110	.2350	.4890	.961	.9800
.35	.644	1.0000	8.164	.3500	.3500	.644	.8030
.40	.682	.6515	4.073	.4960	.2606	.4443	.6660
.45	.704	.4390	2.168	.6800	.1975	.3092	55.60
.50	.707	.3024	1.210	.9090	.1512	.2138	.4620
.55	.693	.2104	.6954	1.1980	.1158	.1460	.3820
.60	.644	.1477	.4103	1.5600	.0887	.0951	.3085

TABLE III.

DURAND PROPELLER NO. 7

$\frac{p}{D}=0.7$

$\frac{V}{nD}$	η	C_2	C_3	$\sqrt{\frac{1}{C_3}}$ F	$\frac{V}{nD}C_2$ C_4	$\eta\, C_2$	$\sqrt{\eta}\, C_2$
0.20	0.395	8.5250	213.1000	0.0685	1.7050	3.3680	1.8350
.25	.473	4.3910	70.2600	.1193	1.0980	2.0780	1.4430
.30	.538	2.5410	28.2400	.1880	.7625	1.3670	1.1680
.35	.597	1.5860	12.9500	.2780	.5550	.9470	.9730
.40	.650	1.0420	6.5130	.3920	.4166	.6770	.8230
.45	.695	.7123	3.5180	.5330	.3206	.4950	.7040
.50	.730	.5009	2.0040	.7070	.2504	.3660	.6050
.55	.755	.3606	1.1920	.9160	.1983	.2720	.5220
.60	.772	.2630	.7306	1.1700	.1578	.2030	.4510
.65	.778	.1945	.4604	1.4740	.1264	.1513	.3890
.70	.767	.1440	.2939	1.8450	.1008	.1105	.3325
.75	.737	.1287	.2288	2.0930	.0965	.0948	.3080

TABLE IV.

DURAND PROPELLER NO. 3.

$\frac{p}{D}=0.9.$

$\frac{V}{nD}$	η	C_2	C_3	$\sqrt{\frac{1}{C_3}}$ F	$\frac{V}{nD}C_2$ C_4	$\eta\, C_2$	$\sqrt{\eta C_2}$
0.20	0.353	10.6000	265.0000	0.0614	2.1220	3.7400	1.9350
.25	.425	5.5000	87.9700	.1066	1.3750	2.3380	1.5280
.30	.487	3.2220	35.8000	.1672	.9670	1.5700	1.2530
.35	.544	2.0460	16.7000	.2450	.7153	1.1130	1.0630
.40	.594	1.3750	8.5940	.3410	.5500	.8160	.9040
.45	.638	.9648	4.7640	.4585	.4342	.6160	.7850
.50	.679	.6976	2.7910	.5990	.3488	.4740	.6880
.55	.713	.5168	1.7090	.7650	.2844	.3685	.6070
.60	.744	.3912	1.0870	.9590	.2348	.2910	.5400
.65	.768	.3004	.7112	1.1860	.1953	.2305	.4800
.70	.788	.2338	.4772	1.4470	.1637	.1843	.4290
.75	.803	.1825	.3244	1.7570	.1369	.1465	.3830
.80	.809	.1432	.2238	2.1130	.1146	.1158	.3400
.85	.809	.1114	.1542	2.5480	.0947	.0901	.3000
.90	.805	.0863	.1065	3.0650	.0777	.0695	.2640
.95	.786	.0659	.0730	3.7000	.0626	.05185	.2280
1.00	.752	.0498	.0498	4.4850	.0498	.03748	.1936

TABLE V.

DURAND PROPELLER NO. 82.

$\frac{p}{D}=1.1$

$\frac{V}{nD}$	η	C_2	C_3	$\sqrt{\frac{1}{C_3}}$ F	$\frac{V}{nD}C_2$ C_4	$\eta\, C_2$	$\sqrt{\eta C_2}$
0.25	0.374	6.3560	101.7000	0.0991	1.5890	2.3750	1.5420
.30	.436	3.7330	41.4800	.1553	1.1200	1.6260	1.2750
.35	.490	2.4000	19.6000	.2260	.8395	1.1750	1.0840
.40	.540	1.6480	10.3000	.3120	.6590	.8890	.9430
.45	.584	1.1820	5.8370	.4140	.5320	.6900	.8310
.50	.628	.8753	3.5010	.5340	.4376	.5500	.7420
.55	.668	.6636	2.1940	.6750	.3650	.4430	.6660
.60	.704	.5120	1.4220	.8380	.3072	.3610	.6010
.65	.734	.3999	.9465	1.0280	.2600	.2940	.5420
.70	.757	.3149	.6427	1.2470	.2205	.2380	.4880
.75	.778	.2496	.4437	1.5000	.1872	.1940	.4400
.80	.796	.1985	.3102	1.7950	.1588	.1580	.3980
.85	.811	.1586	.2195	2.1340	.1348	.1283	.3580
.90	.823	.1267	.1564	2.5300	.1141	.1040	.3227
.95	.832	.1014	.1124	2.9800	.0964	.0843	.2885
1.00	.834	.0811	.0811	3.5100	.0811	.0677	.2603
1.05	.830	.0646	.0586	4.1300	.06785	.0536	.2316
1.10	.817	.0509	.0421	4.8700	.05600	.0416	.2040
1.15	.794	.0396	.0300	4.7700	.04553	.0315	.1775

TABLE VI.

DURAND PROPELLER NO. 113.

$\frac{p}{D}=1.3$

$\frac{V}{nD}$	η	C_s	C_3	$\sqrt{\frac{1}{C_s}}$ F	$\frac{V}{nD}C_s$ C_4	$\eta\ C_s$	$\sqrt{\eta\ C_s}$
0.25	0.311	7.7380	123.800	0.0898	1.96350	2.4050	1.5500
.30	.369	4.5230	50.250	.1410	1.35700	1.6690	1.2920
.35	.425	2.8850	23.560	.2061	1.00800	1.2240	1.1070
.40	.478	1.9690	12.310	.2850	.78700	.9410	.9700
.45	.529	1.4150	6.987	.3780	.63680	.7480	.8650
.50	.577	1.0560	4.224	.4880	.52800	.6090	.7810
.55	.619	.8138	2.690	.6100	.44750	.5040	.7100
.60	.655	.6390	1.775	.7510	.38350	.4190	.6470
.65	.685	.5080	1.202	.9120	.33030	.3480	.5900
.70	.711	.4082	.8331	1.0950	.28580	.2900	.5390
.75	.735	.3304	.5874	1.3160	.24800	.2430	.4930
.80	.756	.2696	.4213	1.5406	.21570	.2040	.4520
.85	.775	.2210	.3059	1.8100	.18780	.1715	.4140
.90	.791	.1820	.2247	2.1080	.16380	.1440	.3800
.95	.806	.1502	.1664	2.4500	.14270	.1210	.3480
1.00	.819	.1240	.1240	2.8400	.12400	.1016	.3186
1.05	.831	.1024	.0929	3.2800	.10750	.0850	.2915
1.10	.838	.0844	.0698	3.7700	.09280	.0708	.2660
1.15	.840	.0696	.0526	4.3700	.08005	.0585	.2420
1.20	.837	.0573	.0398	5.0100	.06875	.0481	.2195
1.25	.826	.0469	.0300	5.7700	.05630	.03875	.1970
1.30	.803	.0381	.0225	6.6600	.04573	.0306	.1750

TABLE VII.

VARIATION OF MAXIMUM EFFICIENCY η_m AND CORRESPONDING

$\frac{V}{nD}$ with $\frac{p}{D}$

Durand Propeller No.	$\frac{p}{D}$	η_m	$\frac{V}{nD}$
139	0.3	0.524	0.28
11	0.5	.708	.48
7	0.7	.778	.65
3	0.9	.810	.83
82	1.1	.834	1.00
113	1.3	.840	1.17

REFERENCES.

The Speed and Power of Ships. D. W. Taylor.

Experimental Research on Air Propellers. W. F. Durand and W. P. Lesley, National Advisory Committee for Aeronautics, Technical Reports Nos. 14, 30, 64, 109, and 141.

The General Efficiency Curve for Air Propellers. W. S. Diehl, National Advisory Committee for Aeronautics, Technical Report No. 168.